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# STRATEGIC BEHAVIOR UNDER COMPLETE IGNORANCE: APPROVAL AND CONDORCET-TYPE VOTING RULES

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Abstract. Usually strategic misrepresentation of preferences in order to manipulate social choice functions is studied under the standard common knowledge assumption. In this paper, we introduce the completely opposite hypothesis of manipulation under complete ignorance. Our goal is to give an answer to the following question : do there still exist any strategic voting opportunities, even if individuals do not have any information about others' preferences ? We provide an exhaustive answer for Condorcet-type and a class of approval voting type SCFs.

## 1. INTRODUCTION

During the last three decades, a large literature has been devoted to the manipulation of social choice mechanisms. Gibbard (1973) and Satterthwaite (1975) proved that there does not exist any social choice procedure selecting a single alternative, which is simultaneously non dictatorial and immune to manipulation. Following these two authors, many contributors have been interested in checking the robustness of this result in different directions. For example, the equilibrium concept used by Gibbard and Satterthwaite has been relaxed in a variety of ways (see for example Pattanaik 1976, Sengupta 1978, or Mbih 1995 among others) ; another direction has been the computation of the frequency of manipulation opportunities (e.g. Lepelley and Mbih 1994). There is however one line of enquiry which has received very little attention; it concerns the possibility for individuals participating to the collective choice procedure not to be completely informed on preferences expressed by other individuals. A notable exception is due to Sengupta (1980) ; supposing a set of three possible alternatives, he shows, for a wide class of procedures based on pairwise comparisons and satisfying some very attractive properties, that in order to manipulate it may be sufficient for some individual to only know either the most preferred or the least preferred alternative of each of the other individuals.

Now what does the phrase "not completely informed" mean ? At least two interpretations are possible. First, one can think of situations, before voting takes place in a committee for instance, where some individual $i$ knows that some other individual $j$ prefers alternative $x$ to alternative $y$ , but she does not know anything else about $j$'s preferences, namely how $j$ ranks $x$ or $y$ vis--vis any other alternative. The second interpretation is based on a probabilistic approach; given a voting rule, considering all configurations of preferences and assuming that every individual only knows his own preferences, it is possible to compute the frequency of election of each alternative; then every individual can interpret these frequencies as a distribution of probabilities over the set of alternatives;

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more concretely, before every real election, polls are generally intended to give the percentage of votes expected by every candidate, and it is theoretically possible, from the polls data, to evaluate the probability of each candidate of being elected. Voters can then use this information to choose the preferences they are going to express.

In this paper, we shall not be concerned with the probabilistic approach; we refer the reader to Andjiga, Mbih and Moyouwou (2003) for an introduction. We shall limit our analysis to the first interpretation of incomplete information, and more precisely, we focuse on the extreme case in which no individual knows anything about other individuals' rankings of alternatives. Then a rational individual chooses a preference relation that better serves her interests, that is a preference that permits her in all circumstances to secure an outcome at least as good as the outcome secured by her sincere preference relation.

We examine the manipulablity of the classes of voting rules. The first one contains special versions of approval voting in which a fixed number of alternatives are "approved of" by the voters ; approval voting is often said to be one of the least manipulable voting rules (see Brams and Fishburn 1984, or Yunfeng, Yue and Chen 1996) ; our results show that for the special versions we study, strategic behavior is still possible under complete ignorance, though under very binding constraints. The second set of rules we are interested in is a class of voting procedures based on the Condorcet principle : an alternative beating every other alternative in pairwise majority contests is called a Condorcet winner ; and whenever it exists, a Condorcet winner is chosen by the voting procedure.

The paper is organized as follows : section 2 introduces notations and definitions ; section 3 is concerned with results and proofs, and section 4 concludes the paper with some general remarks.

## 2. NOTATIONS AND DEFINITIONS

Let $N = \{1, 2, ..., i, ..., n\}$ be a set of $n$ individuals ($n \geq 2$) and $A = \{a_1, a_2, ..., a_j, ..., a_m\}$ be a set of $m$ alternatives ($m \geq 3$). $L$ will denote the set of all possible linear orders on $A$ (i.e. complete, reflexive, antisymmetric and transitive binary relation on $A$) and $R = a_1a_2a_3...$ the linear order such that $a_1$ is the best alternative according to relation $R$, $a_2$ the second best, $a_3$ the third best and so on. A *profile* is an $n$-tuple $R^N = (R^1, ..., R^i, ...R^n)$ of individual relations, one for each individual. $L^N$ will denote the set of all possible profiles. A *contingency* is a profile from which the preference relation of some individual $i$ has been removed, that is an $(n-1)-$tuple $R^{-i} = (R^1, ..., R^{i-1}, R^{i+1}, ...R^n)$; and the set of all contingencies for individual $i$ will be denoted $L^{-i}$. Profile $R^N$ can now be rewritten $R^N = (R^i, R^{-i})$.

Given a non empty subset $B$ of $A$, $lex(B)$ will be the alternative $a_j$ in $B$ with the least index $j$.

**Definition 2.1.** *An SCF $f$, a mapping from $L^N$ into $A$, is manipulable under complete ignorance if there exists some individual $i$, with sincere preference $R^i$ and strategic preference $Q^i$ such that :*

*(a) for all $H^{-i} \in L^{-i}, f(Q^i, H^{-i})R^i f(R^i, H^{-i})$*

*(b) and for some $H^{-i} \in L^{-i}, f(Q^i, H^{-i}) \neq f(R^i, H^{-i})$*

When $(a)$ and $(b)$ in Definition 2.1 hold, individual $i$ has incentive to submit the false preference $Q^i$ to the social choice procedure $f$, rather than his sincere preference $R^i$, no matter the contingency expressed by other individuals.

For example, let $f$ be the $SCF$ which for every profile selects the less preferred alternative according to individual 1's preference relation. Then $f$ is manipulable under complete ignorance : as compared with his sincere strategy, it will always be advantageous to individual 1 to express a false ordering, with her most preferred alternative ranked last.

## 3. Results on manipulation under complete ignorance

3.1. **Approval voting when individuals select a fixed number of alternatives.** According to the general approval principle (see Brams and Fishburn, 1983), each individual has to select one or more alternatives she most prefers, and given a tie-breaking mechanism, the social outcome is chosen from the set of the most selected alternatives. The reader can verify that such a social choice procedure is not an *SCF*; one way to define an *SCF* consists in fixing a number $k$ $(1 \leq k \leq m-1)$ of alternatives each individual has to select.

Let $n_k(a_j, R^N)$ be the number of individuals who rank $a_j$ among their $k$ most preferred alternatives, and define $AP_k(R^N) = \{x \in A : n_k(x, R^N) \geq n_k(y, R^N) \text{ for all } y \in A\}$. Note that $AP_k(R^N) \neq \emptyset$ for all $R^N$.

**Definition 3.1.** *An SCF $f$ is a $k-$approval voting rule ($AV_k$ rule) if for all profile $R^N$, $f(R^N) \in AP_k(R^N)$.*

$AV_1$ and $AV_{m-1}$ rules are versions of the well-known plurality and antiplurality rules respectively. The $AV_k$ rule with ties broken in favor of $lex\left[AP_k(R^N)\right]$ is called *lexicographic* $AV_k$ rule and is denoted $LAV_k$.

**Proposition 3.1.** *1) No $AV_1$ rule is manipulable under complete ignorance if and only if $n \geq 3$.*

*2) $LAV_1$ is not manipulable under complete ignorance.*

*Proof.* 1) Let $f$ denote any $AV_1$, and assume $x, y$ and $z$ are three distinct alternatives. Consider some individual $i$ and let $i$'s sincere preference relation be $R^i = x...$, and let $Q^i = y...$ be some other preference relation for $i$. In order to prove that $LAV_1$ is manipulable under complete ignorance for $n \geq 3$, it will be sufficient to show that there exists some contingency $H^{-i}$ such that $f(R^i, H^{-i}) \neq f(Q^i, H^{-i})$ and $f(R^i, H^{-i})\, R^i f(Q^i, H^{-i})$. We shall distinguish two cases :

(a) $n \geq 3$ and odd. Consider two disjoint subsets $N_1$ and $N_2$ of $N - \{i\}$ with $\frac{n-1}{2}$ members each and construct contingency $H^{-i}$ such that for all $i \in N_1$, $H^i = xy...$ and for all $i \in N_2$, $H^i = yx...$ .

(b) $n \geq 4$ and even. Consider some $j \neq i$ and two disjoint subsets $N_1$ and $N_2$ of $N - \{i, j\}$ with $\frac{n}{2} - 1$ members each, and construct contingency $H^{-i}$ such that $H^j = z...$, for all $i \in N_1$, $H^i = xy...$ and for all $i \in N_2$, $H^i = yx...$ .

Then, in both cases, for all $AV_1$'s $f$, $f(R^i, H^{-i}) = x$, $f(Q^i, H^{-i}) = y$, and $xR^iy$. Therefore no $AV_1$ is manipulable under complete ignorance. It remains to show that for $n = 2$, there exists some $AV_1$ which is manipulable under complete ignorance. Suppose $n = 2$, and let $g$ be an $AV_1$ defined as follows :

$$
\begin{aligned}
g(R^1, R^2) &= a_j \text{ for all } a_j \text{ if } a_2 \text{ is top in } R^1 \text{ and } a_j \text{ is top in } R^2 \\
g(R^1, R^2) &= a_2 \text{ if } a_1 \text{ is top in } R^1 \text{ and } a_2 \text{ is top in } R^2 \\
g(R^1, R^2) &= a_1 \text{ if } a_1 \text{ is top in } R^1 \text{ and } a_2 \text{ is not top in } R^2 \\
\text{and } g(R^1, R^2) &= LAV_1\left(R^1, R^2\right) \text{ if } a_j \text{ is top in } R^1 \text{ and } a_j \notin \{a_1, a_2\}.
\end{aligned}
$$

Clearly $g$ is manipulable under complete ignorance, by replacing a sincere preference $R^i = a_2 a_1...$ with some other preference $Q^i = a_1...$ .

2) From 1), if $n \geq 3$, $LAV_1$ is not manipulable under complete ignorance. For $n = 2$, consider some individual $i$; it will be sufficient, as above, to show that there exists some contingency $H^j$ such that $f(R^i, H^j) \neq f(Q^i, H^j)$ and $f(R^i, H^j)\, R^i f(Q^i, H^j)$. Let $R^i = a_k...$ and $Q^i = a_h...$ . Choose $H^j = a_k...$ if $h < k$ and $H^j = a_h...$ if $h > k$. It clearly follows that $LAV_1$ is not manipulable under complete ignorance. □

**Proposition 3.2.** *Assume* $2 \leq k \leq m-1$.

*1) $LAV_k$ is manipulable under complete ignorance if* $k > \frac{1+m(n-1)}{n}$.

*2) No $AV_k$ is manipulable under complete ignorance if and only if* $k \leq \frac{1+m(n-1)}{n}$.

*Proof* : 1) Suppose $k > \frac{1+m(n-1)}{n}$. Without loss of generality suppose individual 1's sincere preference relation is $R^1 = a_2a_3...a_ka_1a_{k+1}...a_m$ and consider $Q^1 = a_2a_3...a_ka_{k+1}...a_ma_1$. We shall show that $LAV_k$ is manipulable under complete ignorance by submitting $Q^1$ instead of $R^1$.

Let $A_k(R)$ be the set of the $k$ first most preferred alternatives according to linear order $R$. For any contingency $H^{-1}$, we then can write

$$A = \left[\underset{i\neq 1}{\cap} A_k(H^i)\right] \cup \left[\underset{i\neq 1}{\cup}\left[A - A_k(H^i)\right]\right].$$

Clearly $\underset{i\neq 1}{\cap} A_k(H^i)$ and $\underset{i\neq 1}{\cup}\left[A - A_k(H^i)\right]$ are disjoint, hence

$$Card\left[\underset{i\neq 1}{\cap} A_k(H^i)\right] = Card A - Card\left[\underset{i\neq 1}{\cup}\left[A - A_k(H^i)\right]\right],$$

and therefore

$$Card\left[\underset{i\neq 1}{\cap} A_k(H^i)\right] \geq Card A - \sum_{i\neq 1} Card\left[A - A_k(H^i)\right]. \tag{3.1}$$

For each $i \in N - \{1\}$, $Card\left[A - A_k(H^i)\right] = m - k$, then from (3.1) and the fact that $k > \frac{1+m(n-1)}{n}$ can be rewritten $k - 1 > (n-1)(m-k)$ we must have :

$$Card\left[\underset{i\neq 1}{\cap} A_k(H^i)\right] \geq m - (n-1)(m-k) > m - (k-1) \tag{3.2}$$

Inequality (3.2) implies that

$$B = \{a_2, a_3, ..., a_k\} \cap \left[\underset{i\neq 1}{\cap} A_k(H^i)\right] \neq \emptyset.$$

It follows that

$$LAV_k(Q^1, H^{-1}) = lex(B), \text{ and } LAV_k(R^1, H^{-1}) \in \{lex(B), a_1\}.$$

Thus $LAV_k(Q^1, H^{-1}) R^1 LAV_k(R^1, H^{-1})$. And moreover, for all contingencies $H^{-1}$ such that $a_1 \in A_k(H^i)$ for every $i \in N - \{1\}$, $LAV_k(R^1, H^{-1}) = a_1$. And finally we conclude that individual 1 can manipulate $LAV_k$ under complete ignorance.

2) Suppose $k \leq \frac{1+m(n-1)}{n}$. First of all, note that this can be rewritten

$$k - 1 \leq (n-1)(m-k) \tag{3.3}$$

Consider individual 1's sincere preference $R^1$ and some other preference $Q^1$. It is clear that if $A_k(R^1) = A_k(Q^1)$, then individual 1 cannot change the social outcome of an $AV_k$ rule by submitting $Q^1$ instead of $R^1$. Now suppose that $A_k(R^1) \neq A_k(Q^1)$. Since $A_k(R^1)$ and $A_k(Q^1)$ have the same cardinality, we must have $A_k(R^1) - A_k(Q^1) \neq \emptyset$ and $A_k(Q^1) - A_k(R^1) \neq \emptyset$. Let $x \in A_k(R^1) - A_k(Q^1)$ and $y \in A_k(Q^1) - A_k(R^1)$ and define two integers $q$ and $r$ in the following way : $k-1=q(m-k)+r$ with $0\leq r<m-k$. As a consequence of (3.3), we must have $q\leq n-1$ if $r=0$ and $q+1\leq n-1$ if $r>0$. Construct $A_k(R^1)-\{x\}=\underset{2\leq i\leq q+2}{\cup}E_i$ where $E_2,E_3,...,E_{q+2}$ are disjoint subsets of $A_k(R^1)-\{x\}$ such that $cardE_i=m-k$ if $2\leq i\leq q+1$ and $cardE_{q+2}=r$.

And consider some contingency $H^{-1}$ such that :

$$\left\{\begin{array}{l}(a)\text{ for all } i\in N-\{1\},\ \{x,y\}\subseteq A_k(H^i)\\(b)\text{ for all } i\in\{2,3,...,q+2\},\ E_i\subseteq A-A_k(H^i)\end{array}\right.\tag{3.4}$$

Such a contingency can be obtained when every individual in $N-\{1\}$ ranks $x$ first and $y$ second, and each $i=2,3,...,q+2$ does not rank any alternative in $E_i$ among her $k$ most preferred alternatives. From (3.4), $AP_k(R^1,H^{-1})=\{x\}$ and $AP_k(Q^1,H^{-1})=\{y\}$; and therefore for all $AV_k$ rule $f$, $f(R^1,H^{-1})=x$ and $f(Q^1,H^{-1})=y$. Since $xR^1y$, it follows that individual 1 cannot manipulate $f$ under complete ignorance by submitting $Q^1$ instead of $R^1$.

When $k>\frac{1+m(n-1)}{n}$, there exist some $AV_k$ rules (e. g. $LAV_k$) which are manipulable under complete ignorance, as shown above. □

The statements below are straightforward consequences of the results above.

**Corollary 3.1.** *1) No $AV_2$ is manipulable under complete ignorance.*

*2) $LAV_k$ is manipulable under complete ignorance if and only if $k>\frac{1+m(n-1)}{n}$.*

*3) Suppose $2\leq k<m-1$. If $LAV_k$ is manipulable under complete ignorance, then so is $LAV_{k+1}$.*

*4) $LAV_{m-1}$ is manipulable under complete ignorance if and only if $m\geq n+2$.*

*5) If $m\leq n+1$, then no $AV_k$ rule is manipulable under complete ignorance.*

Table 1 and Table 2 summarize these results. It appears that, roughly speaking, profitable misrepresentation of preferences under complete ignorance is possible only when the number of alternatives is greater than the number of voters. This is usually the case in small committees dealing for example with multi-candidate applications, like in then case of recruitment of lecturers in universities. But with large electorates, like in political elections, this will never occur.

3.2. **SCFs based on Condorcet principle.** Given a profile $R^N$, the set $C(R^N)$ of Condorcet winners of $R^N$ is the set of all alternatives that are preferred to any other alternative by at least half of the number of individuals.

**Definition 3.2.** *$f$ is a Condorcet-type SCF ($CT-SCF$) if for every profile $R^N$, $f(R^N)\in C(R^N)$ whenever $C(R^N)$ is a non empty subset of $A$.*

It is well-known that $C(R^N)$ can be empty or contain more than one alternative for $n$ even. We shall call *lexicographic* $CT-SCFs$, denoted $LCT-SCFs$, the $CT-SCFs$ that always break ties in favor of $lex\left[C(R^N)\right]$.

**Proposition 3.3.** *No $CT-SCF$ is manipulable under complete ignorance if and only if $n$ is odd.*

*Proof .* (a) First suppose $n$ is odd. Without loss of generality consider individual 1's sincere preference $R^1$ and some other preference $Q^1$. Let $\{x,y\}\subseteq A$ be such that $xR^1y$ and $yQ^1x$, $N_1$ and $N_2$ be two disjoint subsets of $N-\{1\}$ with $\frac{n-1}{2}$ members each, and $H^{-1}$ be some contingency such that for all $i\in N_1$, $H^i=xy...$ and for all $i\in N_2$, $H^i=yx...$ . Each $CT-SCF$ $f$ verifies $f(R^1,H^{-1})=x$ and $f(Q^1,H^{-1})=y$. Thus $f$ is not manipulable under complete ignorance by submitting $Q^1$ instead of $R^1$.

(b) Now suppose $n$ is even. Consider $\{x,y,z\} \subseteq A$ and $f$ any $CT-SCF$ where ties are broken as follows :
$(T1)$ $f(R^N) = x$ if $C(R^N) = \{x,y\}$ and $n(x,y,R^N) \geq \frac{n}{2}$ where $n(x,y,R^N) = Card\{i \in N : xR^iy\}$.
$(T2)$ $f(R^N) = y$ if $C(R^N) = \{x,y\}$ and $n(x,y,R^N) < \frac{n}{2}$.
$(T3)$ $f(R^N) = y$ if $y \in C(R^N)$ and $C(R^N) - \{x,y\} \neq \emptyset$.
$(T4)$ $f(R^N) = lex\left[C(R^N) - \{x\}\right]$ if $y \notin C(R^N)$ and $C(R^N) - \{x\} \neq \emptyset$.
$(T5)$ $f(R^N) = x$ if $C(R^N) = \emptyset$, $n(x,y,R^{-1}) = \frac{n}{2} - 1$ and $n(x,a,R^{-1}) \geq \frac{n}{2}$ for all $a \neq x$.
$(T6)$ $f(R^N) = z$ if $C(R^N) = \emptyset$ and $(n(x,y,R^{-1}) \neq \frac{n}{2} - 1$ or $n(x,a,R^{-1}) < \frac{n}{2}$ for some $a \neq x)$.

Consider individual 1's sincere preference $R^1 = xy...z$ and $Q^1 = yx...z$ such that $R^1$ and $Q^1$ differ only on $\{x,y\}$. We shall show that $f$ is manipulable under complete ignorance by submitting $Q^1$ instead of $R^1$.

Every contingency $H^{-1}$ verifies $C(R^1,H^{-1}) - \{x\} \subseteq C(Q^1,H^{-1}) \subseteq C(R^1,H^{-1}) \cup \{y\}$.

Suppose $C(Q^1,H^{-1}) \neq C(R^1,H^{-1})$. One of the following cases holds.
Case 1 : $y \notin C(Q^1,H^{-1})$. Therefore $C(Q^1,H^{-1}) = C(R^1,H^{-1}) - \{x\}$ and $y \notin C(R^1,H^{-1})$. If $C(Q^1,H^{-1}) \neq \emptyset$, then from $(T4)$, $f(Q^1,H^{-1}) = f(R^1,H^{-1}) = lex\left[C(R^1,H^{-1}) - \{x\}\right]$. If $C(Q^1,H^{-1}) = \emptyset$, then $C(R^1,H^{-1}) = \{x\}$ and from $(T5)$, $f(Q^1,H^{-1}) = f(R^1,H^{-1}) = x$.
Case 2 : $y \in C(Q^1,H^{-1})$ and $C(R^1,H^{-1}) - \{x,y\} \neq \emptyset$. From $(T3)$ and $(T4)$, $f(Q^1,H^{-1}) = y$ and $f(R^1,H^{-1}) \neq x$. So $f(Q^1,H^{-1})R^1f(R^1,H^{-1})$.
Case 3 : $y \in C(Q^1,H^{-1})$ and $C(R^1,H^{-1}) - \{x,y\} = \emptyset$. Therefore $C(Q^1,H^{-1}) \subseteq \{x,y\}$. If $C(Q^1,H^{-1}) = \{x,y\}$, then $C(R^1,H^{-1}) = \{x\}$, $f(R^1,H^{-1}) = x$ and from $(T1)$, $f(Q^1,H^{-1}) = x$. But if $C(Q^1,H^{-1}) = \{y\}$, then $C(R^1,H^{-1}) = \{x,y\}$ or $C(R^1,H^{-1}) = \emptyset$. From $(T2)$ and $(T6)$, $f(Q^1,H^{-1}) = y$, and $f(R^1,H^{-1}) = y$ or $f(R^1,H^{-1}) = z$.

For each case above we obtain $f(Q^1,H^{-1})R^1f(R^1,H^{-1})$. Moreover for some contingency $H^{-1}$ such that $\frac{n}{2}$ individuals have preferences $xy...$ and $\frac{n}{2} - 1$ other individuals have preferences $yxz...$, we must have $C(R^1,H^{-1}) = \{x,z\}$ and $C(Q^1,H^{-1}) = \{x,y,z\}$. Then from $(T3)$ and $(T4)$, $f(Q^1,H^{-1}) = y$ and $f(R^1,H^{-1}) = z$ respectively. Hence $f$ is manipulable under complete ignorance by submitting $Q^1$ instead of $R^1$.

**Proposition 3.4.** *No $LCT-SCF$ is manipulable under complete ignorance if and only if $n \neq 2$.*

*Proof :* From Proposition 3.3, no $LCT-SCF$ is manipulable under complete ignorance if $n$ is odd. Now suppose that $n$ is even and $n \geq 4$. Let $R^1$,$Q^1 \in L$, $\{a_j,a_k\} \subseteq A$ such that $j < k$, $a_jR^1a_k$ and $a_kQ^1a_j$; let $N_1$ and $N_2$ be two disjoint subsets of $N - \{1\}$ with $\frac{n}{2} - 1$ and $\frac{n}{2}$ members respectively; consider $H^{-1} \in L^{-1}$ such that for all $i \in N_1$, $H^i = a_ja_k...$ and for all $i \in N_2, H^i = a_ka_j...$ . Since $C(R^1,H^{-1}) = \{a_j,a_k\}$ and $C(Q^1,H^{-1}) = \{a_k\}$, $f(R^1,H^{-1}) = a_j$ and $f(Q^1,H^{-1}) = a_k$ for any $LCT-SCF$. Therefore $f$ is not manipulable under complete ignorance by submitting $Q^1$ instead of $R^1$.

Suppose $n = 2$. The reader can check that there is a unique $LCT-SCF$ $f$ which is manipulable under complete ignorance by submitting $Q^1 = a_2a_3a_1...$ instead of $R^1 = a_2a_1a_3...$ as shown in the table below.

| | individual 2's ranking on $\{a_1,a_2,a_3\}$ | | | | | |
|---|---|---|---|---|---|---|
| | $a_1a_2a_3$ | $a_1a_3a_2$ | $a_2a_1a_3$ | $a_2a_3a_1$ | $a_3a_1a_2$ | $a_3a_2a_1$ |
| $R^1 = a_2a_1a_3...$ | $a_1$ | $a_1$ | $a_2$ | $a_2$ | $a_1$ | $a_2$ |
| $Q^1 = a_2a_3a_1...$ | $a_1$ | $a_1$ | $a_2$ | $a_2$ | $a_2$ | $a_2$ |

□

## 4. Concluding remarks

In this paper, we consider the manipulability of two classes of $SCF$s, when individuals completely ignore how other agents rank alternatives. In that context, we show that Condorcet-based $SCF$s are in the general case immune to strategic manipulation when the number of individuals is odd. And for $LCT - SCF$s, there remain limited opportunities for strategic voting only when the society is reduced to two individuals. For the class of approval voting rules under consideration, the answer is less optimistic: we can find situations in which, by misrepresenting her preferences, some individual can without any risk secure an outcome she prefers to the outcome chosen under sincere preferences. Nevertheless, it is remarkable that, in contrast with the striking negative feature of the Gibbard-Satterthwaite theorem, most of the statements proved in this paper are rather positive. Furthermore, even pathological situations are clearly extreme cases, as shown in the Tables. Now, it would be interesting to extend these results into at least three other distinct - though not disjoint - lines of enquiry: 1) the study of manipulation under complete ignorance for other classes of $SCF$s, like scoring voting methods for example, 2) the comparison of $SCF$s on the basis of the minimum level of information necessary for manipulation, and 3) the study of manipulation under a probabilistic framework.

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## Appendix

Table 1. Upper bound values of $k$ such that no $AV_k$ rule is manipulable under complete ignorance

| $n \downarrow$ $m \rightarrow$ | 3 | 4 | 5 | 15 | 30 | 45 | 60 | 75 | 90 | 105 | 120 | 135 | 150 | 165 | 180 | 195 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2* | 2 | 2 | 3 | 8 | 15 | 23 | 30 | 38 | 45 | 53 | 60 | 68 | 75 | 83 | 90 | 98 |
| 3 | 2 | 3 | 3 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 | 110 | 120 | 130 |
| 4 | 2 | 3 | 4 | 11 | 22 | 34 | 45 | 56 | 67 | 79 | 90 | 101 | 112 | 124 | 135 | 146 |
| 5 | 2 | 3 | 4 | 12 | 24 | 36 | 48 | 60 | 72 | 84 | 96 | 108 | 120 | 132 | 144 | 156 |
| 15 | 2 | 3 | 4 | 14 | 28 | 42 | 56 | 70 | 84 | 98 | 112 | 126 | 140 | 154 | 168 | 182 |
| 30 | 2 | 3 | 4 | 14 | 29 | 43 | 58 | 72 | 87 | 101 | 116 | 130 | 145 | 159 | 174 | 188 |
| 45 | 2 | 3 | 4 | 14 | 29 | 44 | 58 | 73 | 88 | 102 | 117 | 132 | 146 | 161 | 176 | 190 |
| 60 | 2 | 3 | 4 | 14 | 29 | 44 | 59 | 73 | 88 | 103 | 118 | 132 | 147 | 162 | 177 | 191 |
| 75 | 2 | 3 | 4 | 14 | 29 | 44 | 59 | 74 | 88 | 103 | 118 | 133 | 148 | 162 | 177 | 192 |
| 90 | 2 | 3 | 4 | 14 | 29 | 44 | 59 | 74 | 89 | 103 | 118 | 133 | 148 | 163 | 178 | 192 |
| 105 | 2 | 3 | 4 | 14 | 29 | 44 | 59 | 74 | 89 | 104 | 118 | 133 | 148 | 163 | 178 | 193 |
| 120 | 2 | 3 | 4 | 14 | 29 | 44 | 59 | 74 | 89 | 104 | 119 | 133 | 148 | 163 | 178 | 193 |
| 135 | 2 | 3 | 4 | 14 | 29 | 44 | 59 | 74 | 89 | 104 | 119 | 134 | 148 | 163 | 178 | 193 |
| 150 | **2** | **3** | 4 | 14 | 29 | 44 | 59 | 74 | 89 | 104 | 119 | 134 | 149 | 163 | 178 | 193 |
| 165 | **2** | **3** | 4 | 14 | 29 | 44 | 59 | 74 | 89 | 104 | 119 | 134 | 149 | 164 | 178 | 193 |
| 180 | **2** | **3** | 4 | 14 | 29 | 44 | 59 | 74 | 89 | 104 | 119 | 134 | 149 | 164 | 179 | 193 |
| 195 | **2** | **3** | 4 | 14 | 29 | 44 | 59 | 74 | 89 | 104 | 119 | 134 | 149 | 164 | 179 | 194 |

*For $n$=2, $k$ must be different from 1.

Table 2. Lower bound values of $n$ such that no $AV_k$ rule is manipulable under complete ignorance

| $k \downarrow$ $m \rightarrow$ | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| 3 | – | 3 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| 4 | – | – | 4 | 3 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| 5 | – | – | – | 5 | 3 | 3 | 2 | 2 | 2 | 2 | 2 |
| 6 | – | – | – | – | 6 | 4 | 3 | 3 | 2 | 2 | 2 |
| 7 | – | – | – | – | – | 7 | 4 | 3 | 3 | 3 | 2 |
| 8 | – | – | – | – | – | – | 8 | 5 | 4 | 3 | 3 |
| 9 | – | – | – | – | – | – | – | 9 | 5 | 4 | 3 |
| 10 | – | – | – | – | – | – | – | – | 10 | 6 | 4 |
| 11 | – | – | – | – | – | – | – | – | – | 11 | 6 |
| 12 | – | – | – | – | – | – | – | – | – | – | 12 |

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